

Percent of Section II score—25

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Every year, each student in a nationally representative sample is given tests in various subjects. Recently, a random sample of 9,600 twelfth-grade students from the United States were administered a multiple-choice United States history exam. One of the multiple-choice questions is below. (The correct answer is C.)

In 1935 and 1936 the Supreme Court declared that important parts of the New Deal were unconstitutional. President Roosevelt responded by threatening to

- (A) impeach several Supreme Court justices
- (B) eliminate the Supreme Court
- (C) appoint additional Supreme Court justices who shared his views
- (D) override the Supreme Court's decisions by gaining three-fourths majorities in both houses of Congress

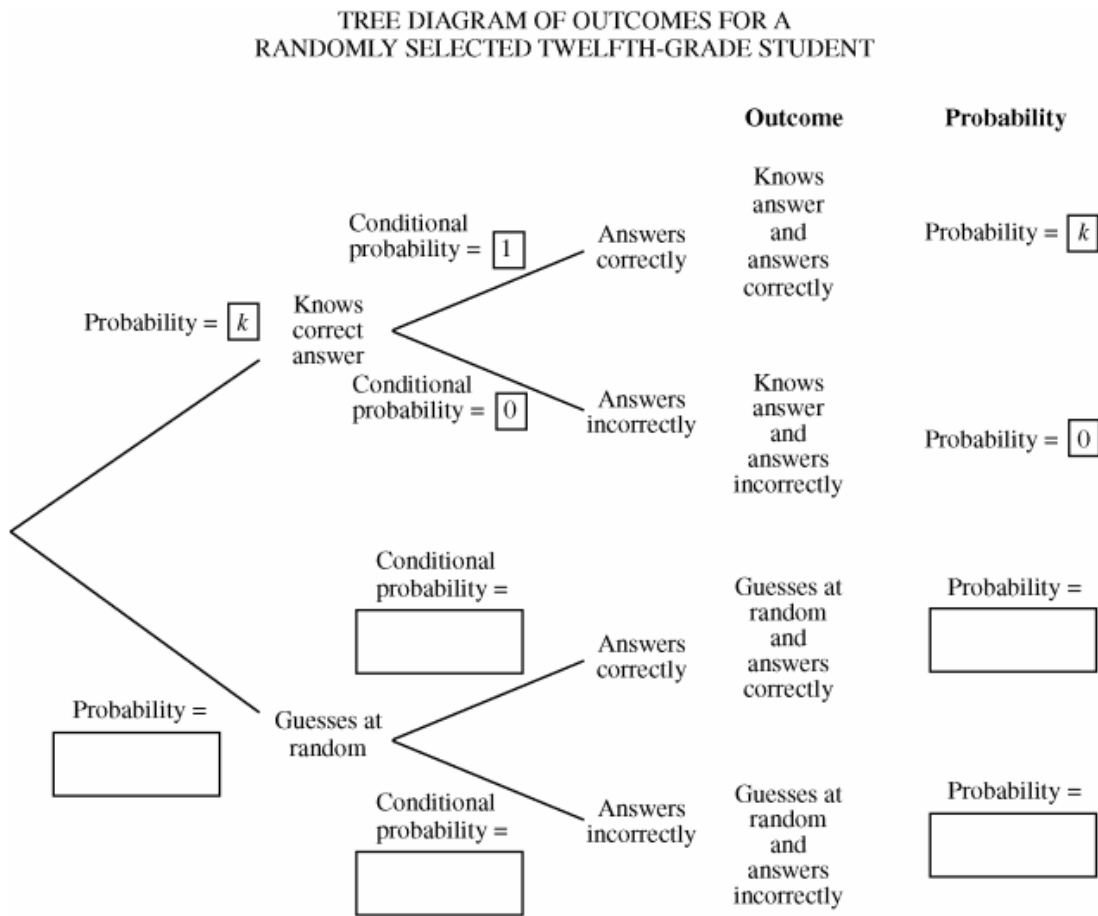
Of the 9,600 students, 28 percent answered the multiple-choice question correctly.

- (a) Let  $p$  be the proportion of all United States twelfth-grade students who would answer the question correctly. Construct and interpret a 99 percent confidence interval for  $p$ .

Assume that students who actually know the correct answer have a 100 percent chance of answering the question correctly, and students who do not know the correct answer to the question guess completely at random from among the four options.

Let  $k$  represent the proportion of all United States twelfth-grade students who actually know the correct answer to the question.

- (b) A tree diagram of the possible outcomes for a randomly selected twelfth-grade student is provided below. Write the correct probability in each of the five empty boxes. Some of the probabilities may be expressions in terms of  $k$ .



- (c) Based on the completed tree diagram, express the probability, in terms of  $k$ , that a randomly selected twelfth-grade student would correctly answer the history question.
- (d) Using your interval from part (a) and your answer to part (c), calculate and interpret a 99 percent confidence interval for  $k$ , the proportion of all United States twelfth-grade students who actually know the answer to the history question. You may assume that the conditions for inference for the confidence interval have been checked and verified.

6. Systolic blood pressure is the amount of pressure that blood exerts on blood vessels while the heart is beating. The mean systolic blood pressure for people in the United States is reported to be 122 millimeters of mercury (mmHg) with a standard deviation of 15 mmHg.

The wellness department of a large corporation is investigating whether the mean systolic blood pressure of its employees is greater than the reported national mean. A random sample of 100 employees will be selected, the systolic blood pressure of each employee in the sample will be measured, and the sample mean will be calculated.

Let  $\mu$  represent the mean systolic blood pressure of all employees at the corporation. Consider the following hypotheses.

$$H_0 : \mu = 122$$

$$H_a : \mu > 122$$

- (a) Describe a Type II error in the context of the hypothesis test.
- (b) Assume that  $\sigma$ , the standard deviation of the systolic blood pressure of all employees at the corporation, is 15 mmHg. If  $\mu = 122$ , the sampling distribution of  $\bar{x}$  for samples of size 100 is approximately normal with a mean of 122 mmHg and a standard deviation of 1.5 mmHg. What values of the sample mean  $\bar{x}$  would represent sufficient evidence to reject the null hypothesis at the significance level of  $\alpha = 0.05$  ?

The actual mean systolic blood pressure of all employees at the corporation is 125 mmHg, not the hypothesized value of 122 mmHg, and the standard deviation is 15 mmHg.

- (c) Using the actual mean of 125 mmHg and the results from part (b), determine the probability that the null hypothesis will be rejected.
- (d) What statistical term is used for the probability found in part (c) ?
- (e) Suppose the size of the sample of employees to be selected is greater than 100. Would the probability of rejecting the null hypothesis be greater than, less than, or equal to the probability calculated in part (c) ? Explain your reasoning.